2 Electric Field : Electric Dipole

Syllabus : Concept of Electric Field, $\vec{E} = \vec{F}/q_0$. Electric Lines of force. Electric Dipole. Electric Field at a point on the axis and on perpendicular bisector of a dipole. Electric dipole moment. Torque on a dipole in a uniform electric field.

1. Field of a Physical Quantity : Scalar and Vector Fields

A physical quantity can be expressed as a continuous function of position in space. The function is called a 'field' of the physical quantity.

Fields are of two kinds : scalar field and vector field.

Scalar Field : When a 'scalar' physical quantity is expressed from point to point in space by a continuous point function $\phi\,(x, y, z)$ which gives the value of the quantity at each point, then, the function ϕ is a 'scalar field'. The space variation of temperature throughout a room, expressed by a function, say, $T\,(x, y, z)$, constitutes a 'temperature field'. Similarly, the space variation of pressure throughout a fluid constitutes a 'pressure field'. These are 'scalar fields' because temperature and pressure are scalar quantities.

Vector Field : When a 'vector' physical quantity is expressed from point to point in space by a continuous point function $\vec{A}\,(x, y, z)$, then the function $\vec{A}\,(x, y, z)$ is a 'vector field'. The space variation of velocity in a fluid, and the variation of gravitational, magnetic and electric intensities in space are examples of 'vector fields'. A vector field in a 3-dimensional space is essentially 3 different scalar fields, one each the directions of $\hat{i}$, $\hat{j}$ and $\hat{k}$ unit vectors. Thus,

$$\vec{A}\,(x, y, z) = A_x\,(x, y, z)\,\hat{i} + A_y\,(x, y, z)\,\hat{j} + A_z\,(x, y, z)\,\hat{k}$$

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Coulomb's law describes the forces between charges as a direct and instantaneous interaction known as "action at a distance" which is represented as

charge ⇌ charge

However much apart the charges may be, they affect one another across the distance separating them. We can look at the above interaction slightly differently by introducing an electric field as an intermediary between the charges. Then, the interaction will be represented as

charge ⇌ electric field ⇌ charge

That is, the first charge sets up an electric field and the second charge placed in the electric field of the first charge experiences a force given by Coulomb's law.

Let us consider an electric charge q located in space. If we bring another charge q_0 near the charge q, then the charge q_0 experiences a force (attraction or repulsion) due to the charge q. The force experienced by q_0 is said to be due to the "electric field" set up by the charge q. Thus, the charge q creates a physical condition, called 'field', in the space surrounding it. Any other charge when placed in this field experience an electrical force.

The charge q is called the "source-charge" and the charge q_0 is called the "test-charge". The source charge may be a point-charge, a group of point-charges or a continuous distribution of charges. Further, the *test-charge must be vanishingly small.*

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Intensity (or Strength) of Electric Field : To determine the intensity of electric field at a point, we place a small positive test-charge at that point and measure the electrostatic force acting on the test-charge. *The intensity of electric field at a point in space is the ratio of the force acting on a unit test-charge placed at that point in space.* (In this definition it has been assumed that the test-charge q_0 is so small that on placing it in the electric field there is no change in the field.) Thus, if $\vec{F}$ be the force acting on a test-charge q_0 placed at a point in an electric field, then the intensity $\vec{E}$ of the field at that point is given by

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

Force $\vec{F}$ is a vector quantity and charge q_0 is a scalar quantity. Hence, intensity of the field $\vec{E}$ is also a vector quantity and its direction is the same as the direction of the force $\vec{F}$ i.e. the direction in which the 'positive' charge placed in the field tends to move.

Force on a charge in an Electric Field : If the intensity of electric field $\vec{E}$ at a point in space be known then we can determine the force $\vec{F}$ acting on a charge q placed at that point by the following equation

$$\vec{F} = q\vec{E}.$$

SI unit of Electric Field : The SI unit of force $\vec{F}$ is 'newton' (N) and that of charge q_0 is 'coulomb' (C). Hence, according to the above equation, the SI unit of electric field $\vec{E}$ is 'newton/coulomb' (N/C). Another SI unit of electric field is 'volt/metre' (V/m).

Dimensions of Electric Field : The unit of electric field $\vec{E}$ is

$$\frac{\text{newton}}{\text{coulomb}} = \frac{\text{kg} \times \text{metre second}^{-2}}{\text{ampere} \times \text{second}} = \text{kg metre second}^{-3}\ \text{ampere}^{-1}.$$

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$\therefore$ dimensions of electric field are $[M\,L\,T^{-3}A^{-1}]$.

3. Electric Field Intensity due to a Point-charge

Suppose an isolated point-charge of $+q$ coulomb is placed at a point O (Fig. 1) in a medium whose dielectric constant is K. There is a point P, distant r metre from O at which the intensity of the field is to be determined. For this, let us assume that a test-charge q_0 is placed at the point P. According to Coulomb's law the electric force acting on q_0 is $F = \frac{1}{4\pi\varepsilon_0 K}\frac{q\,q_0}{r^2}$

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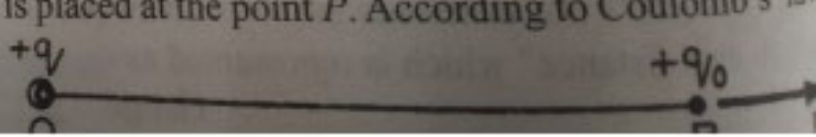

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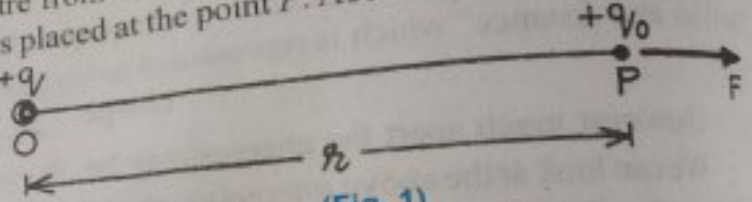

(Fig. 1)

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2} \text{ newton/coulomb.}$$

For vacuum (or air) $K = 1$.

$$\therefore \quad \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ newton/coulomb,}}$$

where $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$ newton metre^2/coulomb^2.

In vector notation $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$,

where $\hat{r}$ is a unit vector along from the source-charge towards the test-charge.

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The force F on the test-charge q_0 due to the source-charge $+q$ is a force of repulsion directed along OP (Fig. 1). Hence the direction of electric field E at the point P will also be along OP (that is, away from the charge $+q$).

If the source-charge at O were $-q$, then the direction of the electric field E at the point P would have been along PO (that is, towards the charge $-q$).

Electric Field due to a Group of Point-Charges : Superposition : If there are n point-charges q_1, q_2, q_3, ... q_n then each of them will produce the same intensity at any point which it would have produced in the absence of other point-charges. Hence, by the principle of superposition, the intensity of the field $\vec{E}$ at a point P due to all the n charges will be equal to the *vector* sum of the intensities $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, ... produced by the separate charges at P :

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i ,$$

where r_i is the distance of the point P from the charge q_i.

If there is a continuous distribution of charge, then the summation in the above expression will be replaced by integration. If the charge is distributed on a surface, then the electric intensity at a point P is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma}{r^2} \hat{r}\, dS ,$$

where r is the distance of the point P from a surface element dS and σ is the surface charge density.

Similarly, if the charge is distributed in a volume, then

$$\vec{E} = \frac{1}{} \int \frac{\rho}{} \hat{} \, dV$$

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$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho}{r^2} \hat{r}\, dV ,$$

where ρ is the volume charge density.

4. Electric Field due to a Circular Loop of Charge or Charged Ring (Optional)

Let O (Fig. 2) be the centre of a circular ring of radius a and having a charge q (say, positive) uniformly distributed over it. Its

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \frac{q}{(a^2 + x^2)^{3/2}}$$

This is the expression for the electric intensity due to a *uniformly-charged* ring at an axial point. The intensity is along the axis, pointing outwards (if charge on the ring is positive). The intensity perpendicular to the axis is zero.

Special Cases : (i) If the point P lies at the centre of the ring, then $x = 0$ and so

$$E = 0,$$

as is obvious by symmetry. A test-charge at the centre of the ring would be pushed or pulled equally in all directions in the plane of the ring and would not experience a net force.

(ii) If the point P is at a *large* distance from the ring ($x >> a$), then a^2 can be ignored compared to x^2 in the above expression. Then, we have

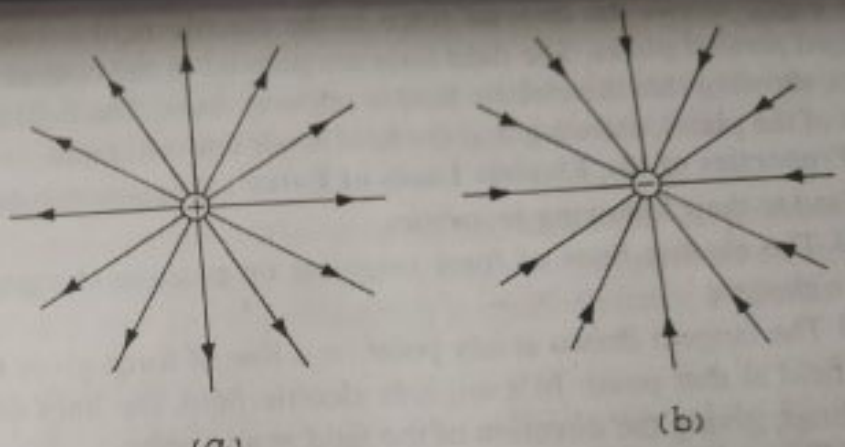
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}.$$

This is the expression for the electric intensity due to a point-charge q at a distance x from it. Thus, *for points on the axis at distances much larger than the radius of the ring, the ring behaves like a point-charge.*

5. Electric Lines of Force

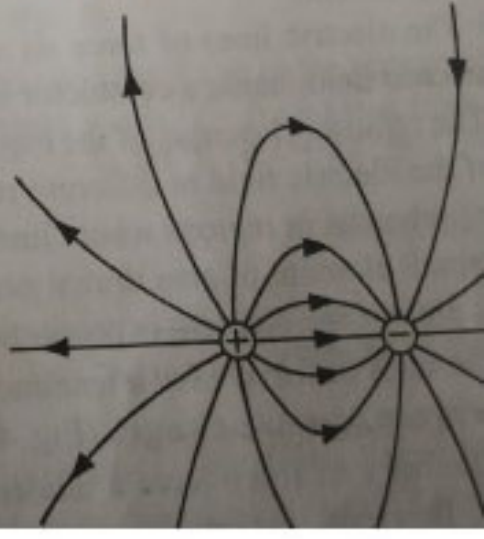
A charge placed in an electric field experiences an electrostatic force. If the charge be free, then it will move in the direction of the force. If the direction of the force is continuously changing, then the direction of motion of the charge will also continuously change, that is, it will move along a curved path. The path of a free positive charge in an electric field is called 'electric line of force'. Hence, *an electric line of force is that imaginary smooth curve drawn in an electric field along which a free, isolated positive charge moves. The tangent drawn at any point on the electric line of force gives the direction of the force acting on a positive charge placed at that point.* An electric field can conveniently be visualized by lines of force.

In Fig. 3 (a) are drawn lines of force in the electric field produced by an isolated positive point-charge. A positive test-charge left free at any point near this point-charge would experience a repulsive force acting radially outwards, and the test-charge would move in that direction. Hence the lines of force of a positive point-charge are straight, directed radially outwards. Conversely, in the electric field of an isolated negative point-charge, the positive test-charge would move radially inwards. Hence, the lines of force are straight, directed radially inwards (Fig. 3 b).



(Fig. 3)

In Fig. 4, the lines of force are drawn in the field produced by two equal and opposite charges at a small distance (electric dipole). These lines originate on the positive charge and terminate on the negative charge. The concentration of lines is greatest in the region between the charges, where the electric field is strongest.



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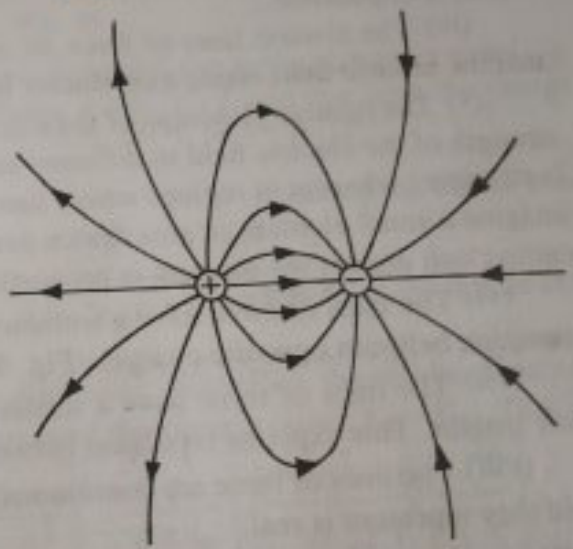
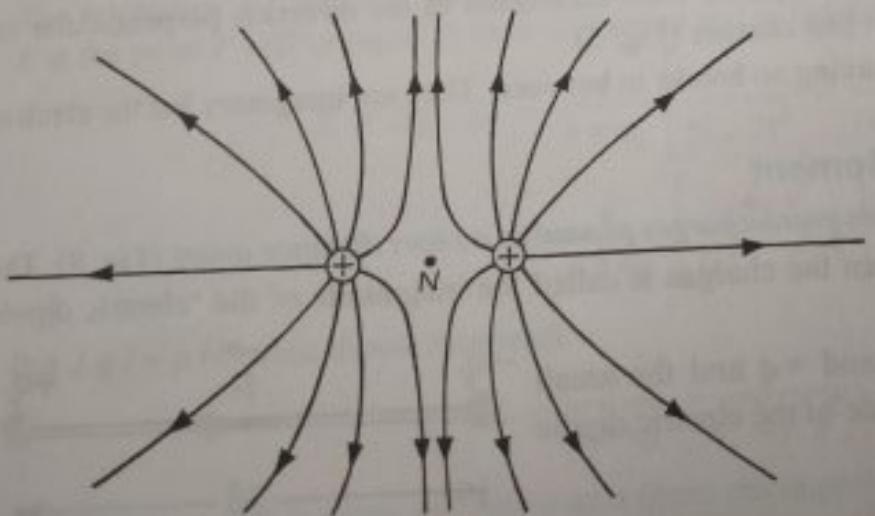


(Fig. 4)

In Fig. 5, lines of force are drawn in the field produced by two equal and similar (positive) charges. At the mid-point N of the line joining the two charges, the field produced by one charge is equal and opposite to the field produced by the other charge. Therefore, at this point the resultant field is zero, as shown by absence of lines of force around N. This point is called the 'neutral point'. If a test-charge be placed at this point, the tendency to move in any direction. As we



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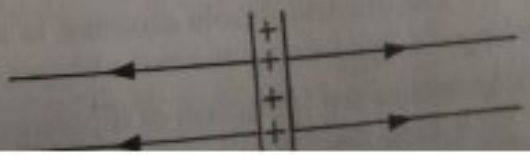

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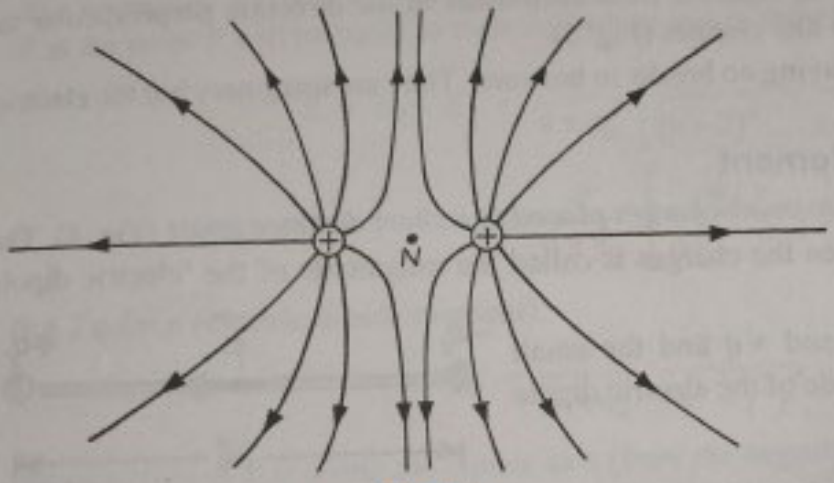

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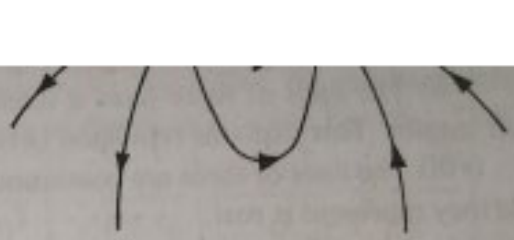

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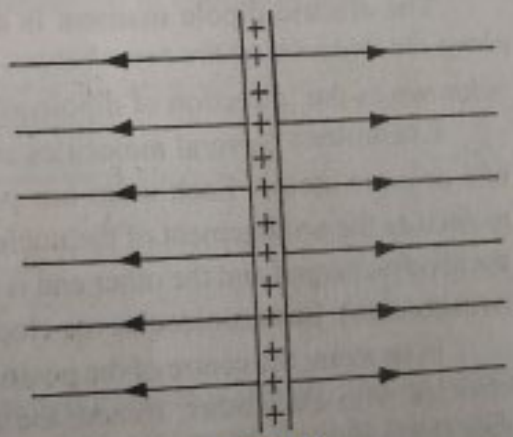
In Fig. 6, are drawn the lines of force due to a long line of positive electric charges. Fig. 5 shows that, in the regions above and below the middle of the two charges, the lines of force are almost parallel in the plane of the figure. If the collection of two charges be extended to a long line of closely spaced positive charges, then in the region close to the middle of the line, the resulting line of force would be parallel, as shown in Fig. 6.



(Fig. 6)

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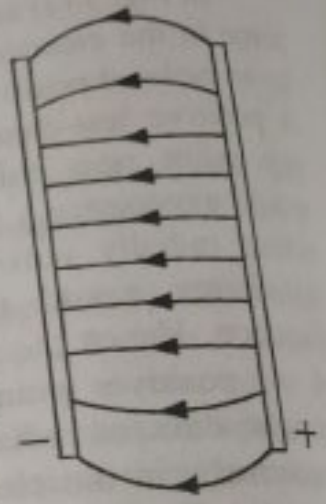
Fig. 7 shows the lines of force in the electric field between two oppositely-charged parallel plates. The field lines are parallel in the central region between the plates, showing that the electric field is *uniform* there. The field lines "fringe" at the edges of the plates, showing that the field is *not* uniform there.



(Fig. 7)

Properties of the Electric Lines of Force : The above pictures of the lines of force lead to their following properties :

(i) The electric lines of force originate on positive charges and terminate on negative charges.

(ii) The tangent drawn at any point on a line of force gives the direction of the electric field at that point. In a *uniform* electric field, the lines of force are parallel straight lines, giving the direction of the field at any point.

(iii) Two lines of force cannot intersect at a common point. For, if they did, it would mean two different directions of the electric field at the point of intersection, which is impossible.

(iv) The electric lines of force *do not* pass through a conductor. This indicates that the electric field inside a conductor is always zero.

(v) The relative closeness of the electric lines of force in different regions of space indicates the rel strength of the electric field in different regions. In regions, where lines of force are closer, the electric s stronger ; whereas in regions where lines of force are farther apart, the field is weaker . If at some po magine a small element of area drawn perpendicular to the lines of force there, the number of lines o utting unit area of the element is proportional to the strength of the electric field there.

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(vi) The lines of force have a tendency to contract in length like a stretched elastic string. This explains attraction between opposite charges (Fig. 4).

(vii) The lines of force have a tendency to separate from each other in the direction perpendicular to their lengths. This explains repulsion between like charges (Fig. 5).

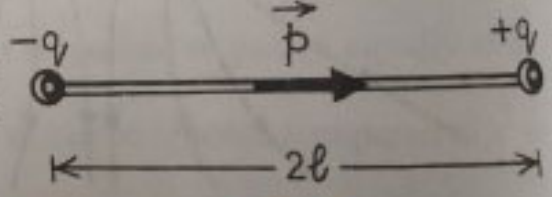
(viii) The lines of force are continuous, having no breaks in between. They are imaginary but the electric field they represent is real.

6. Electric Dipole : Electric Dipole Moment

An electric dipole is a pair of equal and opposite point-charges placed at a short distance apart (Fig. 8). The product of one charge and the distance between the charges is called the magnitude of the 'electric dipole moment' p.

Suppose the charges of dipole are $-q$ and $+q$ and the small distance between them is $2l$. Then, the magnitude of the electric dipole moment is

$$p = q \times 2l = 2ql.$$



(Fig. 8)

The electric dipole moment is a 'vector' $\vec{p}$ whose direction is *along the line joining the two charges, pointing from the negative charge to the positive charge* (Fig. 8). This is known as the 'direction of dipole axis'. The SI unit of electric dipole moment is 'coulomb-metre' (C m).

Examples : Several molecules as HCl, H_2O, etc. are electric dipoles. We ...

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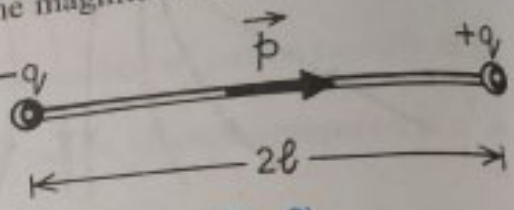

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Examples : Several molecules as HCl, H_2O, etc. are electric dipoles. We know that each molecule has two or more atoms. Each atom has positively-charged nucleus and negatively-charged electrons. In some molecules the arrangement of the nuclei and the electrons of the atoms is such that one end of the molecule is positively-charged and the other end is equally negatively-charged (although the 'net' charge on the molecule remains zero). Such molecules are electric dipoles.

In an atom, the centre of the positive charges (nucleus) and the centre of the negative charges (electrons) coincide with each other. Hence, the atom is *not* a dipole. But if the atom is placed in an electric field then the positive and negative centres are displaced relative to each other and the atom becomes a dipole

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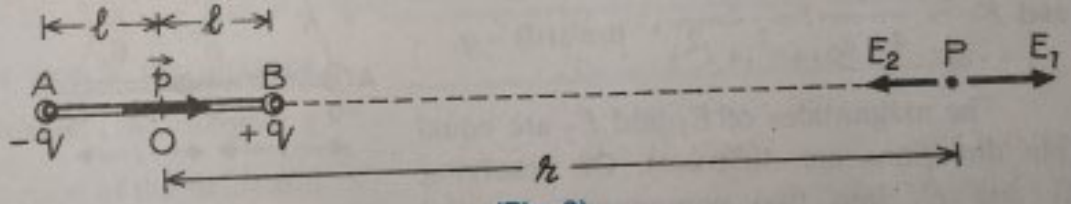
Lines of Force : The lines of force of an electric dipole are drawn in Fig. 4. They originate at the positive pole and terminate at the negative pole.

We consider two aspects of an electric dipole, namely : the nature of the electric field created by a dipole; and the force (and torque) acting on a dipole when placed in an external electric field.

7. Electric Field due to an Electric Dipole

In this connection two positions are important : (i) end-on position, (ii) broad-side-on position.

(i) Intensity of the Electric Field at a point on the Axis of a Dipole : Suppose an electric dipole AB (Fig. 9) made up of two charges of $-q$ and $+q$ coulomb having a small distance $2l$ metre between them is situated in vacuum (or air). Let P be a point on the axis at a distance r metre from the mid-point O of the dipole. We have to determine the intensity of the electric field at P.



(Fig. 9)

Let E_1 and E_2 be the magnitudes of the intensities of the electric field at P due to the charges $+q$ and $-$ of the dipole respectively. The distance of the point P from the charge $+q$ is $(r-l)$ and that from the charg $-q$ is $(r+l)$. Therefore

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2},$$ (along the d e ax

and

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2},$$ (opposite to the o e a

The intensities E_1 and E_2 are along the same line in opposite directions. Therefore, the resulta nten E at the point P will be equal to their difference and in the direction of dipole axis (since $E_1 > E_2$) hat

$$\frac{q}{} \left[\frac{1}{} - \frac{1}{} \right] \quad \frac{q}{} \left[\frac{(r+l)^2 - (r-l)}{} \right.$$

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The intensities E_1 and E_2 are along the same line in opposite [illegible] direction of dipole axis (since $E_1 > E_2$). That is, E at the point P will be equal to their difference and in the direction of dipole axis (since $E_1 > E_2$). That is,

$$E = E_1 - E_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)^2 - (r-l)^2}{(r^2-l^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right].$$

But $2ql = p$ (electric dipole moment).

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\,\frac{2pr}{(r^2-l^2)^2}.$$

The direction of **E** *is along the dipole axis (from the negative charge towards the positive charge).*

If r is very large compared to $2l$ ($r >> 2l$), then l^2 may be neglected in comparison to r^2. The last expression may now be written as

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{2p}{r^3} \text{ newton / coulomb.}$$

In vector notation :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{2\vec{p}}{r^3}.$$

(ii) Intensity of the Electric Field at a point on the Equatorial Line of a Dipole : Now, suppose that the point P is situated on the right-bisector of the dipole AB at a distance r metre from its mid-point O

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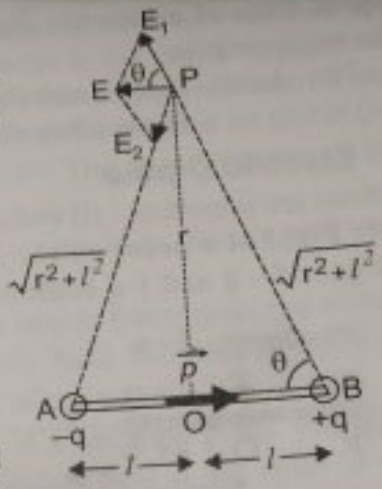
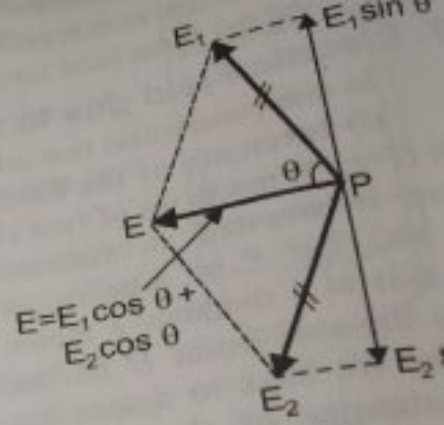
(Fig. 10). Again, let E_1 and E_2 be the magnitudes of the intensities of the electric field at P due to the charges $+q$ and $-q$ of the dipole respectively. The distance of P from each charge is $\sqrt{r^2 + l^2}$. Therefore,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}, \text{ away from } +q$$

and $$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}, \text{ towards } -q.$$



(a)



(Fig. 10)

The magnitudes of E_1 and E_2 are equal (but directions are different). On resolving E_1 and E_2 into two components parallel and perpendicular to AB, the components perpendicular to AB ($E_1 \sin\theta$ and $E_2 \sin\theta$) cancel each other (because they are equal and opposite), while the components parallel to AB ($E_1 \cos\theta$ and $E_2 \cos\theta$), being in the same direction, add up (Fig. 10 b). Hence the resultant intensity of field at the point P is

$$E = E_1\cos\theta + E_2\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}$$

Now, from fig. 10 (a),

$$\cos\theta = \frac{BO}{BP} = \frac{l}{(r^2 + l^2)^{1/2}}.$$

$$\therefore E = \frac{1}{}\frac{q}{}\frac{2l}{} = \frac{1}{}\frac{2ql}{}$$

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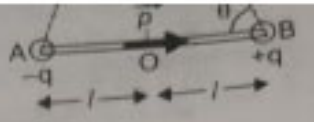
and $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}$, towards $-q$.



(a)

(b)

(Fig. 10)

The magnitudes of E_1 and E_2 are equal (but directions are different). On resolving E_1 and E_2 into two components parallel and perpendicular to AB, the components perpendicular to AB ($E_1 \sin\theta$ and $E_2 \sin\theta$) cancel each other (because they are equal and opposite), while the components parallel to AB ($E_1 \cos\theta$ and $E_2 \cos\theta$), being in the same direction, add up (Fig. 10 b). Hence the resultant intensity of electric field at the point P is

$$E = E_1\cos\theta + E_2\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}2\cos\theta.$$

Now, from fig. 10 (a), $\cos\theta = \frac{BO}{BP} = \frac{l}{(r^2+l^2)^{1/2}}$.

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2ql}{(r^2+l^2)^{3/2}}.$$

But $2ql = p$ (moment of electric dipole).

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+l^2)^{3/2}}.$$

The direction of electric field E is 'antiparallel' to the dipole axis.

If r is very large compared to $2l$ ($r >> 2l$), then l^2 may be neglected in comparison to r^2. Then, the electric intensity at the point P due to the dipole is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \text{ newton / coulomb.}$$

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In vector notation : $\vec{E} = \frac{1}{\ldots}\vec{p}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}$$

Now, from fig. 10 (a), $\cos\theta = \frac{BO}{BP} = \frac{l}{(r^2+l^2)^{1/2}}$.

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2ql}{(r^2+l^2)^{3/2}}.$$

But $2ql = p$ (moment of electric dipole).

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The direction of electric field E is 'antiparallel' to the dipole axis.

If r is very large compared to $2l$ ($r >> 2l$), then l^2 may be neglected in comparison to r^2. Then, the electric intensity at the point P due to the dipole is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \text{ newton / coulomb.}$$

In vector notation :

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}. \quad \text{...(ii)}$$

Minus sign is put because $\vec{E}$ is antiparallel to $\vec{p}$.

Comparing eq. (i) and (ii), we see that at large distances the intensity of the electric field on an axial point is double in magnitude but opposite in direction, than the intensity at the same distance on the equatorial line.

Further, in comparing the fields of a point-charge and of a dipole, the $1/r^2$ dependence is replaced by $1/r^3$ dependence.

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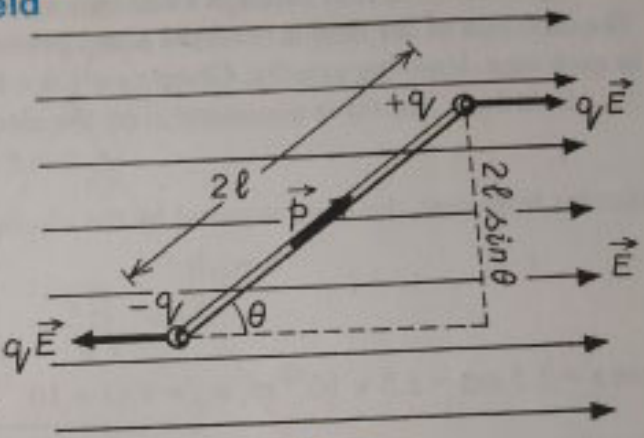
8. Torque on a Dipole in a Uniform Electric Field

When an electric dipole is placed in an external uniform electric field, a couple acts upon the dipole. This couple tends to align the dipole in the direction of the field. This is called the 'restoring couple'.

Let us consider an electric dipole placed in a *uniform* electric field $\vec{E}$ in a region of space (shown by 'parallel' lines of force in Fig. 11), the dipole moment vector $\vec{p}$ making an angle θ with the field $\vec{E}$. Let $-q$ and $+q$ be the charges forming the dipole and $2l$ the distance between them. Due to the electric field $\vec{E}$, the charge $+q$ experiences a force $q\ \vec{E}$ (in the direction of the field) and the charge $-q$ experiences an equal and opposite force $q\ \vec{E}$ (opposite to the field). Since these two forces are equal and opposite, *the net translatory force on the dipole in 'uniform' electric field is zero; therefore there will be to translatory motion of the dipole in a uniform electric field.*



(Fig. 11)

However, the forces $q\ \vec{E}$, $q\ \vec{E}$ act at different points and form a couple which tends to set the dipole parallel to the field $\vec{E}$. The moment of this restoring couple is known as the 'torque' $\vec{\tau}$ on the dipole. The magnitude of the torque (= force × perpendicular distance) is given by

$$\tau = q\,E\,(2\,l \sin\theta) = 2\,q\,l\,E \sin\theta$$

or

$$\boxed{\tau = p\,E \sin\theta \quad \textbf{newton-metre,}}$$

where p ($= 2ql$) is the magnitude of the dipole moment.

In vector form :

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}.}$$

In Fig. 11, the torque is perpendicula

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making an angle θ with the field $\vec{E}$. Let $-q$ and $+q$ be the charges forming the dipole and $2l$ the distance between them. Due to the electric field $\vec{E}$, the charge $+q$ experiences a force $q\vec{E}$ (in the direction of the field) and the charge $-q$ experiences an equal and opposite force $q\vec{E}$ (opposite to the field). Since these two forces are equal and opposite, *the net translatory force on the dipole in 'uniform' electric field is zero; therefore there will be to translatory motion of the dipole in a uniform electric field.*

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$$\tau = q E (2 l \sin\theta) = 2 q l E \sin\theta$$

or

$$\boxed{\tau = p E \sin\theta \quad \textbf{newton-metre,}}$$

where p $(= 2 q l)$ is the magnitude of the dipole moment.

In vector form :

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}.}$$

Thus, in a uniform electric field, a dipole feels a torque (but no net force). In Fig. 11, the torque is perpendicular to the page, pointing downwards (right-hand screw rule).

If the dipole be placed perpendicular to the electric field ($\theta = 90°$ or $\sin\theta = 1$), then the torque acting on it will be maximum. If this be τ_{max} then,

$$\tau_{max} = p E$$

or

$$p = \tau_{max} / E.$$

If $E = 1$ newton/coulomb, then $p = \tau_{max}$ coulomb-metre. Hence, *the moment of an electric dipole is the torque acting on the dipole placed perpendicular to the direction of a uniform electric field of intensity 1 newton/coulomb.*

If, however, a dipole is placed in a non-uniform electric field, it experiences a net force in addition to a torque.

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NUMERICAL EXAMPLES

separated from a proton by a distance of 0.53 Å. Calculate the electric field at the